



NOISE ESTIMATION BY USE OF NEIGHBORING DISTANCES IN TAKENS SPACE AND ITS APPLICATIONS TO STOCK MARKET DATA

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We present a method that uses distances between nearest neighbors in Takens space to evaluate a level of noise. The method is valid even for high noise levels. The method has been verified by estimation of noise levels in several chaotic systems. We have analyzed the noise level for Dow Jones and DAX indexes and we have found that the noise level ranges from 25 to 80% of the signal variance.

Keywords: Noise level estimation; stock market; time series.

1. Introduction

It is common that observed data are contaminated by a noise (for a review of methods of nonlinear time series analysis see [Kantz & Schreiber, 1997; Abarbanel, 1996]). The presence of noise can substantially affect invariant system parameters as a dimension, entropy or Lyapunov exponents. In fact Schreiber [1993] has shown that even 2% of noise can mislead a dimension calculation. It follows that the assessment of the noise level can be crucial for estimation of system invariant parameters. Even after performing a noise reduction one is interested to evaluate the noise level in the cleaned data. In the experiment, noise is often regarded as an uncertainty measurement which corresponds to a random variable added to the system's temporary state or to the experimental outcome. This kind of noise

is usually called *measurement* or *additive* noise. Another case is when noise influences the system dynamics, corresponding to the Langevine equation and can be called the *dynamical* noise. The second case is more difficult to analyze because the noise acting at moment t_0 usually changes the trajectory for $t > t_0$. It follows that there is no clean trajectory and instead, an ϵ -shadowed trajectory occurs [Farmer & Sidorowich, 1991]. For real data a signal (e.g. physical experimental data or economic data) is subjected to the mixture of both kinds of noise (measurement and dynamical).

Schreiber has developed a method of noise level estimation [Schreiber, 1993] by evaluating the influence of noise on the correlation dimension of the investigated system. The Schreiber method is valid for rather small gaussian measurement noise and requires values of the embedding dimension d , the

embedding delay τ and the characteristic dimension r spanned by the system dynamics.

Diks [1996] investigated properties of correlation integral with the gaussian kernel in the presence of noise. The Diks method makes use of a fitting function for correlation integrals calculated from time series for different thresholds ε . The function depends on system variables K_2 (correlation entropy), D_2 (correlation dimension), σ (standard noise deviation) and a normalizing constant Φ . These four variables are estimated using the least squares fitting. The Diks method [Yu *et al.*, 2000] is valid for a noise level up to 25% of signal variance and for various measurement noise distributions. The Diks's method needs optimal values of the embedding dimension d , the embedding delay τ and the maximal threshold ε_c .

Cawley *et al.* [Cawley & Hsu, 1992] developed a method of noise *reduction* and they used this method for noise level estimation. The method explored the local-geometric-projection principle and is useful for various noise distributions but rather small noise levels. To use the method one needs to choose a number of neighboring points to be regarded, an appropriate number of iterations as well as optimal parameters values d and τ .

Oltmans *et al.* [Oltmans & Verheijen, 1997] considered the influence of noise on the probability density function $f_n(\varepsilon)$ but they could take into account only a small measurement noise. They used a fit of $f_n(\varepsilon)$ to the corresponding function which was found for small ε . Their fitting function is similar to the probability density distribution that we receive from correlation integrals $(1/N^2)DET_n(\varepsilon)$. The method needs as input parameter values of d , τ and ε_c .

In [Urbanowicz & Holyst, 2003] we presented a method of noise level estimation by coarse-grained correlation entropy (NECE). The crucial point of this method is to fit a proper function to

the estimated correlation entropy. This method does not demand any input parameters like the embedding dimension d or the embedding delay τ . The minimal and maximal values of the threshold parameter ε can be automatically estimated. The NECE method will be used further as the reference method.

In this paper we present another method for evaluation of a noise level. The method makes use of neighboring distances in the embedding space (NEND) and will be introduced in the next section. In the further section we show an application of this method to stock market data. Although it is a common belief, that the stock market behavior is driven by stochastic processes [Voit, 2001; Bouchaud & Potters, 2000; Mantegna & Stanley, 2000] it is difficult to separate stochastic and deterministic components of market dynamics. In fact the deterministic fraction follows usually from non-linear effects and can possess a nonperiodic or even chaotic characteristic [Peters, 1997; Holyst *et al.*, 2001]. With the help of the NEND and NECE methods [Urbanowicz & Holyst, 2004] we try to demonstrate that stock market data are not purely stochastic and a deterministic part can be sometimes dominant.

2. Method of Noise Estimation by Use of Neighboring Distances in Takens Space (NEND)

Let $\{x_i\}$ where $i = 1, 2, \dots, N$ be a time series and $\mathbf{x}_i = \{x_i, x_{i+\tau}, \dots, x_{i+(d-1)\tau}\}$ a corresponding d -dimensional vector constructed in the embedded space where d is an embedding dimension and τ is an embedding delay. The method is based on the assumption that the minimal distance between nearest neighbors is described by the standard deviation of noise. The nearest neighbor is found using the Euclidian norm i.e. the distance is measured using the following formula

$$\text{dis}_{i,j} = \sqrt{(x_i - x_j)^2 + (x_{i-\tau} - x_{j-\tau})^2 + \dots + (x_{i-(d-1)\tau} - x_{j-(d-1)\tau})^2}. \quad (1)$$

The nearest neighbor of the vector \mathbf{x}_n is the vector \mathbf{x}_j such that

$$\{\mathbf{x}_j : \forall_k \mathbf{x}_k, (k \neq j, n), \text{dis}_{n,j} \leq \text{dis}_{n,k}\}. \quad (2)$$

We will assume that the distance between the vector \mathbf{x}_n and its nearest neighbor (dis_n^{NN}) is calculated in a large embedding dimension $d \gg 1$.

For linear systems without a noise the minimal distance between nearest neighbors should decrease with an increasing number of data in time series and for $N \rightarrow \infty$ this distance will tend to zero since the trajectory reaches the final periodic orbit. For deterministic chaotic systems such minimal distances depend on the system entropy but they also tend

to zero for an infinite number of data when the trajectory densely fills the chaotic attractor.

In the case when we add to an observed deterministic trajectory a Gaussian noncorrelated noise the corresponding minimal distance dis_n^{NN} is increased. For a large value of the embedding dimension $d \gg 1$ the distances can be estimated as a standard deviation of a superposition of $2d$ independent stochastic variables, i.e.

$$\text{dis}_n^{\text{NN}} \approx \sqrt{2d}\sigma, \tag{3}$$

where σ is the standard deviation of a noise added to the signal.

The approximation (3) is valid only in limits of very long time series and a large embedding dimension d . If we generate surrogate data $\{\text{surr}_i\}$ by the random shuffling of the original data [Holyst *et al.*, 2001] then this kind of surrogates preserves mean, variance and histogram but removes any determinism in data. The minimal distance between nearest neighbors calculated in an embedded space for surrogate data should be proportional to standard deviation of data.

Now let us define the Noise-To-Signal ratio as the proportion of σ to the standard deviation of data σ_{data}

$$\text{NTS} = \frac{\sigma}{\sigma_{\text{data}}}. \tag{4}$$

In the first step of the method we calculate all distances between nearest neighbors in the original and in the surrogate data. Then we search for the smallest distance for each data set: $\text{dis}_{\min}^{\text{NN}} = \min_n \{\text{dis}_n^{\text{NN}}\}$ and $\text{dis}_{\min}^{\text{surr, NN}} = \min_n \{\text{dis}_n^{\text{surr, NN}}\}$. Using the approximation (3), i.e. the linear dependence of the distance $\text{dis}_{\min}^{\text{NN}}$ on the noise level, we can introduce the output parameter of the method ADET_d [Urbanowicz *et al.*, 2004], which is related to the Noise-To-Signal ratio (NTS) as follows:

$$\text{NTS} \approx \text{ADET}_d \equiv \frac{\text{dis}_{\min}^{\text{NN}}}{\langle \text{dis}_{\min}^{\text{surr, NN}} \rangle}. \tag{5}$$

Here we denoted $\langle \text{dis}_{\min}^{\text{surr, NN}} \rangle$ as an average of m realizations of the surrogate data (m appears as the parameter of the method).

3. Noise Estimation: Examples and Application to Stock Market Data

The NEND method described in the previous section is very simple to use. A drawback of this method is a large error of the estimated noise level

Table 1. Results of the noise level estimation for different systems. In the case of NEND method we used $d = 9$, $m = 20$ and $N = 3000$.

System	NTS	σ	Estimated σ Using NEND	Estimated σ Using NECE
Henon	0	0	0.0 ± 0.05	-0.0023 ± 0.0001
Henon	0.09	0.1	0.05 ± 0.06	0.1 ± 0.0007
Ikedada	0.1	0.07	0.05 ± 0.04	0.07 ± 0.0005
Lorenz	0.43	4	3.7 ± 0.5	4.4 ± 0.4
Lorenz	0.85	15.7	16.6 ± 1	14.9 ± 0.08
Lorenz	0.96	30	32.8 ± 1.5	30.5 ± 0.7
Roessler	0.33	7.4	4.7 ± 1.2	6.4 ± 0.8
Roessler	0.84	15	14.46 ± 0.8	14.7 ± 1.1
Roessler	0.93	33.4	37 ± 1.8	33.8 ± 1

for short time series and too low embedding dimensions d . The estimation error increases if we take a smaller number of random surrogates data that are used for the averaging formula (5). In Table 1 examples of noise estimation by the NEND method are presented in comparison to results of NECE method (see [Urbanowicz & Holyst, 2003]). The estimation error of the NEND method is based on a standard deviation of temporary values of ADET_d for different realizations of surrogate data. One can see that the NEND method, despite its simplicity, works quite well for considered cases. Although the NECE method gives better accuracy as compared to the NEND method but the first method is much more sophisticated and difficult for computer implementation. CPU times needed by computers are comparable for both methods.

Both methods were applied to evaluate the noise levels in stock market data. Here we present results for Dow Jones Industrial Average (DJIA) during the time period 1896–2002 (daily returns, see Fig. 1) and DAX (German Stock Market Index) during the time period 1998–2000 (4 minutes returns, see Fig. 4). Returns x_n are defined as

$$x_n = \ln \left(\frac{P_n}{P_{n-1}} \right), \tag{6}$$

where P_n is the value of an index at the time n . Noise levels for both indexes are in the range $\text{NTS} \approx 0.5\text{--}0.9$ as one can see in Figs. 2, 3, 5 and 6. It follows that considered stock market data are not purely stochastic because the percent of determinism ranges $(1 - \text{NTS}^2) \cdot 100\% \approx 20\text{--}75\%$ and the stochastic part is about 25–80%. In Figs. 2 and 5 we present noise levels ADET_9 calculated with the NEND method for DJIA and DAX indexes, respectively. For the comparison

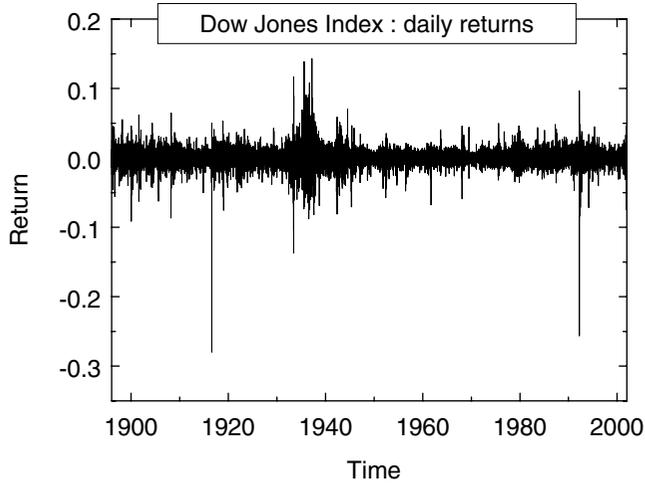


Fig. 1. Plot of daily returns of Dow Jones Index (1896–2002).

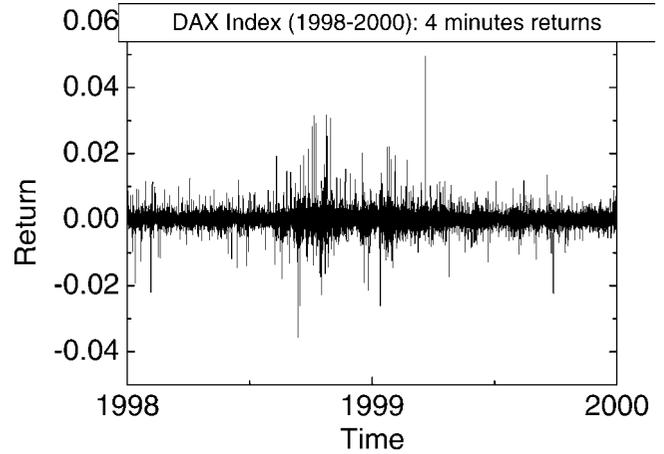


Fig. 4. Plot of 4 minutes returns of DAX Index (1998–2000).

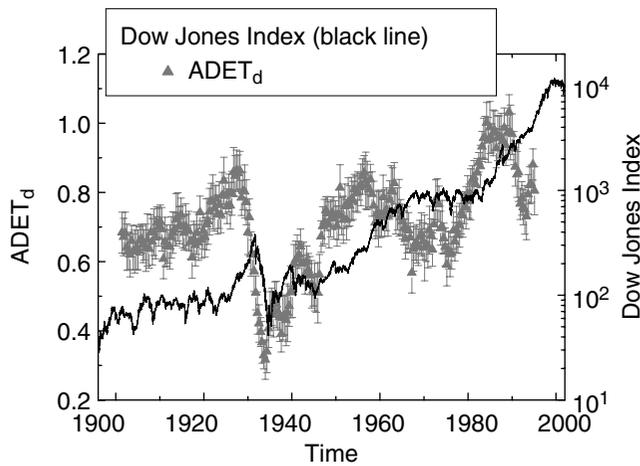


Fig. 2. Plot of noise levels $ADET_d$ for $d = 9$ calculated with the NEND method and the value of Dow Jones Index (1896–2002).

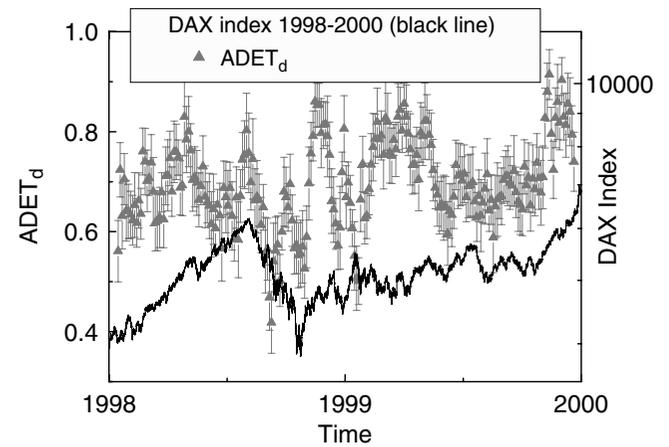


Fig. 5. Plot of noise levels $ADET_d$ for $d = 9$ calculated with the NEND method and the value of DAX Index (1998–2000).

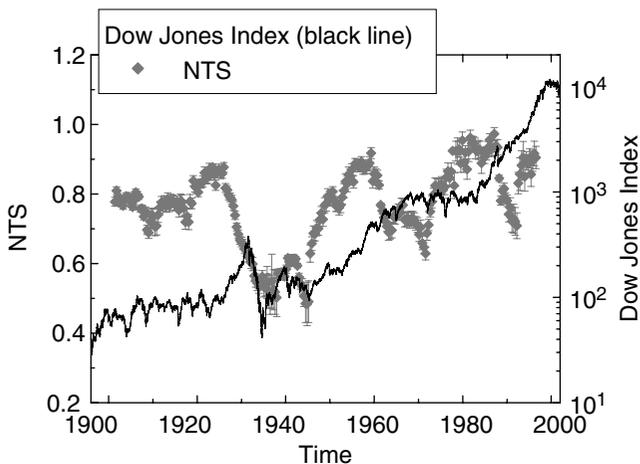


Fig. 3. Plot of noise levels NTS calculated with the NECE method and the value of Dow Jones Index (1896–2002).

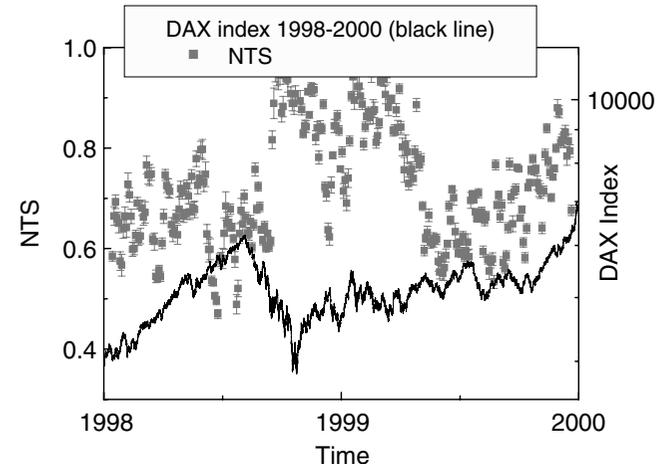


Fig. 6. Plot of noise levels NTS calculated with the NECE method and the value of DAX Index (1998–2000).

in Figs. 3 and 6 we show noise levels NTS calculated with the NECE method. The NEND method and the reference method NECE give similar results. Since both methods are different approaches thus similar results suggest good accuracy of both methods. Some differences in noise level estimations between both methods appear in the periods of increased volatility (1930–1940 for DJIA and from August to November of 1998 for DAX), which suggests that extreme events have different impact in both methods. We think that the NECE method gives more relevant results in high volatility regions and the NEND method underestimates the noise level in such cases.

4. Conclusions

In conclusion we have developed a new method of noise level estimation from time series. The method makes use of the minimal distance between nearest neighbors in Takens space. The method has been tested for several systems and it has brought similar results to the method described in [Urbanowicz & Hołyst, 2003] but it is much easier for computer implementation. The application of the method to stock market data gives the percent of noise ranging from 25 to 80% of signal variance.

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